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Gravitational Evolution of the Large-Scale Density Distribution: The Edgeworth & Gamma Expansions

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Abstract. The gravitational evolution of the cosmic one-point Probability Distribution Function (PDF) can be estimated using an analytic approximation that combines gravitational Perturbation Theory (PT) with the Edgeworth expansion around a Gaussian PDF. We present an alternative to the Edgeworth series based on an expansion around the Gamma PDF, which is more appropriate to describe a realistic PDF. The Gamma expansion converges when the PDF exhibits exponential tails, which are predicted by PT and N-body simulations in the weakly non-linear regime (*i.e*, when the variance, σ^2, is small). We compare both expansions to N-body simulations and find that the Gamma expansion yields a better overall match to the numerical results.

1. Introduction

Combining non-linear perturbation theory with the Edgeworth expansion has largely succeeded in describing the gravitational evolution of the large-scale density PDF in the weakly non-linear regime, for Gaussian initial conditions (Juszkiewicz et al 1995, Bernardeau & Koffman 1995). In principle, the accuracy of this approach is only limited by the order of the (reduced) cumulants, S_J, involved in the Edgeworth expansion. However, the Edgeworth series yields a PDF that is ill-defined. It has negative probability values and assigns non-zero probability to negative densities ($\delta < -1$). Alternatively, we shall introduce the Gamma PDF as the basis for an expansion in orthogonal (Laguerre) polynomials around an arbitrary exponential tail (see Gaztañaga, Fosalba & Elizalde 1999). The proposed Gamma expansion is better suited for describing a realistic PDF, as always yields positive densities and the PDF is effectively positive-definite.

2. Comparison of the expansions with N-body simulations

Figure 1 shows a comparison of the Edgeworth and Gamma expansions with N-body simulations. We measure the PDF in 10 realizations of SCDM, $\Omega = 1$ and $\Gamma = 0.5$, with $L = 180\,h^{-1}$ Mpc and $N = 64^3$ particles and $\sigma_8 = 1$ (Croft & Efstathiou 1994). We find that, up to second order, *both expansions produce*

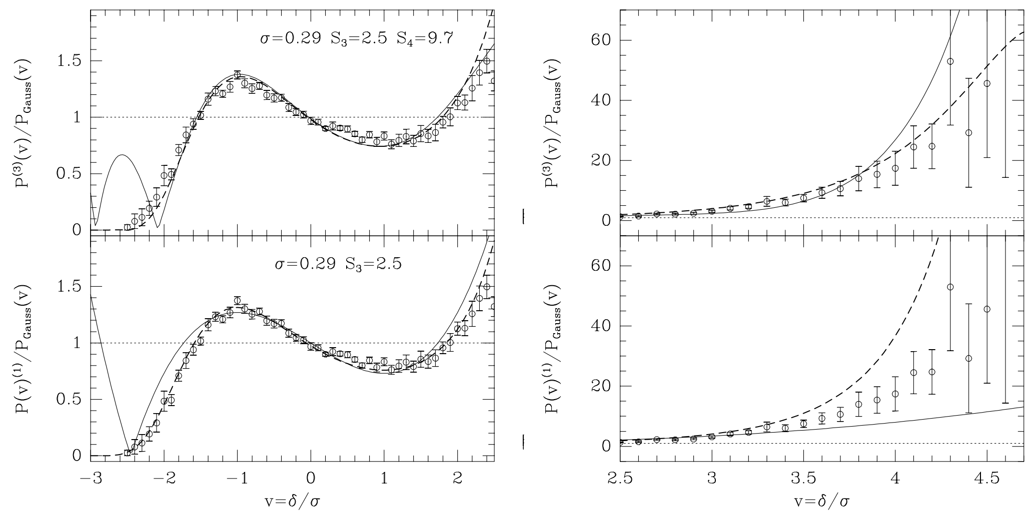

Figure 1. Deviations from the Gaussian PDF for both expansions and in N-body simulations (symbols). The lower (upper) panel displays results for the first (third) order in powers of σ, for different values of the skewness S_3 and kurtosis S_4. The solid line is given by the Edgeworth series while the dashed one shows the Gamma expansion. The left and right panels show different ranges in ν.

very similar results, specially around the peak of the distribution, within the error bars. However, *the Gamma expansion provides a better match to the PDF on the tails.* In particular, the Gamma expansion is in far better agreement with the numerical results for negative values of ν (see left panel) and performs slightly better for the positive tail of the PDF, $\nu \simeq 1 - 5$ (see right panel).

In summary, we propose the Gamma expansion as a useful alternative to the Edgeworth series to model the gravitational evolution of the large-scale density PDF in the weakly non-linear regime. We stress the potential application of the Gamma expansion for modeling other non-Gaussian PDFs, such as those describing the peculiar velocities of galaxies or the temperature anisotropy of the CMB on small scales.

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